



*Tutorials and worked examples for simulation,  
curve fitting, statistical analysis, and plotting.  
<http://www.simfit.org.uk>*

Given  $n$  functions of  $n$  variables in a user-defined model file it is sometimes possible to estimate zeros as long as good starting estimates are input, and a reasonable tolerance factor is provided.

The model file must define a system of  $n$  equations  $f_i$  in  $n$  variables  $x_i$  and the SIMFIT program **usermod** will attempt to locate  $x_1, x_2, \dots, x_n$  such that

$$f_i(x_1, x_2, \dots, x_n) = 0, \text{ for } i = 1, 2, \dots, n.$$

Users must supply good starting estimates by editing the default starting estimates  $y_1, y_2, \dots, y_n$ , or installing a new  $y$  vector from a file, and the accuracy can be controlled by varying  $xtol$ , since the program attempts to ensure that

$$\|x - \hat{x}\| \leq xtol \times \|\hat{x}\|,$$

where  $\hat{x}$  is the true solution, as described for NAG routine C05NBF. Failure to converge will lead to nonzero **IFAIL** values, requiring a re-run with new starting estimates.

From the main SIMFIT menu choose [A/Z] then open program **usermod** and input test file **usermodn\_e.tf4** which defines 9 equations in 9 variables for the following tridiagonal system.

$$\begin{aligned} (3 - 2x_1)x_1 - 2x_2 + 1 &= 0 \\ -x_{i-1} + (3 - 2x_i)x_i - 2x_{i+1} + 1 &= 0, \quad i = 2, 3, \dots, 8 \\ -x_8 + (3 - 2x_9)x_9 + 1 &= 0. \end{aligned}$$

After setting the starting estimates  $y(i) = 0$  for  $i = 1, 2, \dots, 9$  proceed to locate zeros of  $n$  equations in  $n$  variables when the following table will result.

Variable	Value	Function	Value
$x(1)$	-5.7065289E-01	$fvec(1)$	2.5267933E-06
$x(2)$	-6.8162532E-01	$fvec(2)$	1.5688139E-05
$x(3)$	-7.0173246E-01	$fvec(3)$	2.8357029E-07
$x(4)$	-7.0421463E-01	$fvec(4)$	-1.3083878E-05
$x(5)$	-7.0136741E-01	$fvec(5)$	9.8768418E-06
$x(6)$	-6.9186497E-01	$fvec(6)$	6.5557114E-06
$x(7)$	-6.6579418E-01	$fvec(7)$	-1.3053615E-05
$x(8)$	-5.9603414E-01	$fvec(8)$	1.1777047E-06
$x(9)$	-4.1641142E-01	$fvec(9)$	2.9510981E-06

The values displayed at the solution point are as follows.

<b>IFAIL</b>	IFAIL = 0, otherwise re-run.
<b>FNORM</b>	final 2-norm of the residuals.
<b>xtol</b>	Tolerance factor.
<b><math>x(i)</math></b>	Estimates for $x_i$ .
<b><math>fvec(i)</math></b>	$f_i(x)$ values.

As values less than about  $10^{-7}$  are effectively zero compared to 1, this represents a satisfactory outcome since  $f_i(x) \approx 0$  for  $i = 1, 2, \dots, n$  at the solution point.

For reference, the model is as follows.

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%
Example: 9 functions of 9 variables as in NAG C05NBF
      set y(1) to y(9) = -1 or 0 for good starting estimates
f(1)=(3-2x(1))x(1)-2x(2)+1, &, f9=-x(8)+(3-2x(9))x(9)+1
%
9 equations
9 variables
0 parameters
%
begin{expression}
f(1) = (3 - 2y(1))y(1) + 1 - 2y(2)
f(2) = (3 - 2y(2))y(2) + 1 - y(1) - 2y(3)
f(3) = (3 - 2y(3))y(3) + 1 - y(2) - 2y(4)
f(4) = (3 - 2y(4))y(4) + 1 - y(3) - 2y(5)
f(5) = (3 - 2y(5))y(5) + 1 - y(4) - 2y(6)
f(6) = (3 - 2y(6))y(6) + 1 - y(5) - 2y(7)
f(7) = (3 - 2y(7))y(7) + 1 - y(6) - 2y(8)
f(8) = (3 - 2y(8))y(8) + 1 - y(7) - 2y(9)
f(9) = (3 - 2y(9))y(9) + 1 - y(8)
end{expression}

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