



Tutorials and worked examples for simulation,  
curve fitting, statistical analysis, and plotting.

<https://simfit.org.uk>

<https://simfit.silverfrost.com>

The Wilcoxon signed rank test is a sort of nonparametric equivalent of the paired  $t$  test that is used to examine the differences between matched observations in two data sets, say  $X$  and  $Y$ , just assuming a symmetrical, but unspecified, distribution for the paired differences.

To be precise, the user has two samples (i.e. vectors  $X$  with median  $X_{med}$ , and  $Y$  with median  $Y_{med}$ ) with  $n$  observations, and specifies a hypothetical median for the paired differences, say  $D_{med}$ , that generates a vector of signed differences  $d_i = x_i - y_i$  defined by

$$X = (x_1, x_2, \dots, x_n)$$

$$Y = (y_1, y_2, \dots, y_n)$$

$$D = (d_1, d_2, \dots, d_n).$$

Users have to decide whether to include or discard zero differences, and whether to change the default median difference of  $D_{med} = 0$  to  $D_{med} = k$  for some hypothetical  $k$ , then analysis of the signed differences is performed to test if it is reasonable to conclude that either

- both samples have the same population median,
- the population median for sample  $X$  is smaller than that for sample  $Y$ , or
- the population median for sample  $X$  is larger than that for sample  $Y$ .

The test is weak unless large samples are used, and is further weakened by ties within the data, that is, multiple observations with the same value.

From the main SIMFIT menu select [A/Z], choose to open the SIMFIT statistics program **simstat**, then the standard tests option to analyze the test files provided. Choosing a specified zero median and opting to suppress zero differences yields the following results.

#### Wilcoxon paired-sample signed-rank test

Zero differences suppressed, median test value = 0

X-data: test file g08agf . tf1

Y-data: test file g08agf . tf2

Size of data = 8, Number of values suppressed = 0

W 32.00

Z 1.890

$H_0$ : X median = Y median

as null hypothesis against the alternatives:-

$H_1$ : Medians differ

p 0.0547

$H_2$ : X median < Y median

p 0.9805

$H_3$ : X median > Y median

p 0.0273 Reject  $H_0$  at 5% significance level

In this example there were no zero differences to reject, and here  $W$  is the signed ranks test statistic, while  $Z$  is an approximate normal test statistic. Using SIMFIT program **normal**, we find that  $P(Z \geq 1.89) = 0.0294$ . In order to make the interpretation of this test as clear as possible, especially the effect of the value chosen for  $D_{med}$ , the results from sequential analysis of data in SIMFIT test files `wilcoxon . tf1` and `wilcoxon . tf2` using two different values of  $D_{med} = 0$  then  $D_{med} = -2$  are displayed next.

### Wilcoxon paired-sample signed-rank test 1 and test 2

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Median test value = 0  
X-data: test file wilcoxon.tf1  
Y-data: test file wilcoxon.tf2  
Size of data = 50, Number of values suppressed = 0  
W 306.0  
Z -3.195  
 $H_0$ : X median = Y median  
 $H_1$ : Medians differ  
p 0.0011 Reject  $H_0$  at 1% significance level  
 $H_2$ : X median < Y median  
p 0.0005 Reject  $H_0$  at 1% significance level  
 $H_3$ : X median > Y median  
p 0.9995

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Median test value = -2  
W 783.0  
Z 1.400  
 $H_0$ : X median = Y median  
 $H_1$ : Medians differ  
p 0.1629  
 $H_2$ : X median < Y median  
p 0.9200  
 $H_3$ : X median > Y median  
p 0.0815

The following graph shows that, although X and Y do appear to be matched, the difference is mostly around -2, which explains why  $D_{med} = 0$  is rejected, but  $D_{med} = -2$  is not rejected, emphasizing the importance of choosing  $D_{med}$  sensibly on the outcome of this test.

### Data for Wilcoxon Signed Rank Test

