



Tutorials and worked examples for simulation,
curve fitting, statistical analysis, and plotting.

<https://simfit.org.uk>

<https://simfit.silverfrost.com>

This procedure, based on the binomial distribution, is used with dichotomous data, i.e., where an experiment has only two possible outcomes and it is wished to test H_0 : binomial $p = p_0$ for some $0 \leq p_0 \leq 1$. For instance, to test if consecutive outcomes are independent with the same probability, i.e. are Bernoulli trials.

To be precise, you input the number of successes, k , the number of Bernoulli trials, N , and the supposed probability of success, p_0 , then SIMFIT calculates the probabilities associated with k , N , p_0 , and $l = N - k$, including the estimated probability parameter \hat{p} with 95% confidence limits, and the two-tail binomial test statistic. The probabilities for X equal to the number of successes, which can be used for upper-tail, lower-tail, or two-tail testing are

$$\hat{p} = k/N$$

$$P(X = k) = \binom{N}{k} p^k (1-p)^{N-k}$$

$$P(X > k) = \sum_{i=k+1}^N \binom{N}{i} p^i (1-p)^{N-i}$$

$$P(X < k) = \sum_{i=0}^{k-1} \binom{N}{i} p^i (1-p)^{N-i}$$

$$P(X = l) = \binom{N}{l} p^l (1-p)^{N-l}$$

$$P(X > l) = \sum_{i=l+1}^N \binom{N}{i} p^i (1-p)^{N-i}$$

$$P(X < l) = \sum_{i=0}^{l-1} \binom{N}{i} p^i (1-p)^{N-i}$$

$$P(\text{two tail}) = \min(P(X \geq k), P(X \leq k)) + \min(P(X \geq l), P(X \leq l)).$$

Open the SIMFIT main menu then choose [Statistics] followed by [Standard tests] and run the binomial test option using the default parameters to obtain this table of results.

Binomial test analysis 1		
Successes k	5	
Trials N	10	
$l = N - k$	5	
p -theory	0.50000	
p -estimate	0.50000	95% confidence limits = 0.18709, 0.81291
$P(X > k)$	0.37695	
$P(X < k)$	0.37695	
$P(X = k)$	0.24609	
$P(X \geq k)$	0.62305	
$P(X \leq k)$	0.62305	
$P(X > l)$	0.37695	
$P(X < l)$	0.37695	
$P(X = l)$	0.24609	
$P(X \geq l)$	0.62305	
$P(X \leq l)$	0.62305	
Two tail binomial test statistic = 1.00000		

From this table it is obvious that 5 successes in 10 trials is perfectly consistent with a binomial distribution having $N = 10$ and $p = 0.5$. However, consider the results when the number of Bernoulli trials is reduced to 5, where intuition might suggest that five successive heads in coin tossing would suggest a two-headed coin.

Binomial test analysis 2

Successes k	5	
Trials N	5	
$l = N - k$	0	
p -theory	0.50000	
p -estimate	1.00000	95% confidence limits = 0.47818, 1.0000
$P(X > k)$	0.00000	
$P(X < k)$	0.96875	
$P(X = k)$	0.03125	
$P(X \geq k)$	0.03125	
$P(X \leq k)$	1.00000	
$P(X > l)$	0.96875	
$P(X < l)$	0.00000	
$P(X = l)$	0.03125	
$P(X \geq l)$	1.00000	
$P(X \leq l)$	0.03125	

Two tail binomial test statistic = 0.06250

This shows, for example, that the probability of obtaining five successes (or alternatively five failures) in an experiment with equiprobable outcome would not lead to rejection of $H_0 : p = 0.5$ in a two tail test. Note, for instance, that the exact confidence limits for the estimated probability include 0.5. Many life scientists when asked what is the minimal sample size to be used in an experiment, e.g. the number of experimental animals in a trial, would use a minimum of six, since the null hypothesis of no effect would never be rejected with a sample size of five.

The next table illustrates that an experiment with six consecutive successes (or failures) would indicate that the 95% confidence region for the parameter p does not include 0.5, and would provide grounds for rejecting the null hypothesis H_0 : The trials are all independent with $p = 0.5$.

Binomial test analysis 3

Successes k	6	
Trials N	6	
$l = N - k$	0	
p -theory	0.50000	
p -estimate	1.00000	95% confidence limits = 0.54074, 1.0000
$P(X > k)$	0.00000	
$P(X < k)$	0.98438	
$P(X = k)$	0.01563	
$P(X \geq k)$	0.01563	
$P(X \leq k)$	1.00000	
$P(X > l)$	0.98438	
$P(X < l)$	0.00000	
$P(X = l)$	0.01563	
$P(X \geq l)$	1.00000	
$P(X \leq l)$	0.01563	

Two tail binomial test statistic = 0.03125, Reject H_0 at 5% significance level
