



Tutorials and worked examples for simulation,
curve fitting, statistical analysis, and plotting.
<http://www.simfit.org.uk>

Given two matrices A and B , it is frequently necessary to form the product, or the product of the transposes, as an m by n matrix C , where $m \geq 1$ and $n \geq 1$. The options are

$$C = AB, \text{ where } A \text{ is } m \times k, \text{ and } B \text{ is } k \times n,$$

$$C = A^T B, \text{ where } A \text{ is } k \times m, \text{ and } B \text{ is } k \times n,$$

$$C = AB^T, \text{ where } A \text{ is } m \times k, \text{ and } B \text{ is } n \times k,$$

$$C = A^T B^T, \text{ where } A \text{ is } k \times m, \text{ and } B \text{ is } n \times k,$$

as long as $k \geq 1$ and the dimensions of A and B are appropriate to form the product, as indicated.

From the main SIMFIT menu choose [Statistics] followed by [Numerical analysis], and then open the Cholesky factorization procedure and save the lower triangular matrix L to a file. Then use the matrix multiplication procedure from the SIMFIT numerical analysis options to form the product LL^T as shown below.

The current matrix A

4.16	-3.12	0.561	-0.10
-3.12	5.03	-0.83	1.09
0.56	-0.83	0.76	0.34
-0.10	1.09	0.34	1.180

Estimated lower triangular \hat{L} where $A = LL^T$

2.0396078E+00			
-1.5297059E+00	1.6401219E+00		
2.7456259E-01	-2.4998141E-01	7.8874881E-01	
-4.9029034E-02	6.1885642E-01	6.4426613E-01	6.1606334E-01

Estimated product $\hat{A} = \hat{L}\hat{L}^T$

4.1600000E+00	-3.1200001E+00	5.6000000E-01	-1.0000000E-01
-3.1200001E+00	5.0300000E+00	-8.3000000E-01	1.0900000E+00
5.6000000E-01	-8.3000000E-01	7.6000001E-01	3.4000000E-01
-1.0000000E-01	1.0900000E+00	3.4000000E-01	1.1800000E+00

Another example using the singular value decomposition routine, followed by multiplying the calculated U , Σ , and V^T matrices for the simple 4 by 3 matrix indicated shows that, while for exact factors

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1/\sqrt{6} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \\ -1/\sqrt{6} & 0 & -1/\sqrt{2} \\ -2/\sqrt{6} & 0 & 0 \end{pmatrix} \begin{pmatrix} \sqrt{3} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \end{pmatrix},$$

singular value decomposition to yield the calculated factors \hat{U} , $\hat{\Sigma}$ and \hat{V}^T followed by matrix multiplication leads to the following matrix.

The product matrix $\hat{A} = \hat{U}\hat{\Sigma}\hat{V}^T$

1.0000000E+00	-1.5700925E-16	1.2789252E-08
0.0000000E+00	1.0000000E+00	0.0000000E+00
1.2789252E-08	2.2204460E-16	1.0000000E+00
1.0000000E+00	1.5700925E-16	1.0000000E+00

Numbers colored red in the above results tables can be regarded as correct since any digits less than 10^{-7} are due to rounding error and can be taken as zero compared to 1.