



Tutorials and worked examples for simulation,  
curve fitting, statistical analysis, and plotting.  
<http://www.simfit.org.uk>

Quadratic forms often need to be evaluated in data analysis given a  $n$  by 1 vector  $x$  and a  $n$  by  $n$  matrix  $A$ . Frequently the inverse  $A^{-1}$  is required and it is convenient to be able to estimate this interactively.

For instance, in nonlinear optimization or multivariate statistics the following expressions for  $Q_1$  and/or  $Q_2$  are frequently required

$$Q_1 = x^T A x$$
$$Q_2 = x^T A^{-1} x.$$

To evaluate such quadratic forms interactively, open [Statistics] then [Numerical analysis] from the main SIMFIT menu and select the option to evaluate quadratic forms which provides the default test files `matrix.tf3` defining matrix  $A$  and vector `tf3` defining  $x$  as follows

$$A = \begin{pmatrix} 4.16 & -3.12 & 0.56 & -0.10 \\ -3.12 & 5.03 & -0.83 & 1.09 \\ 0.56 & -0.83 & 0.76 & 0.34 \\ -0.10 & 1.09 & 0.34 & 1.18 \end{pmatrix}$$

$$x = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$$

The following table illustrates the output from running the SIMFIT procedure with these default test files.

Title of matrix $A$ Test file matrix.tf3: 4 by 4 positive-definite symmetric matrix Title of vector $x$ Test file vector.tf3: vector with components 1, 2, 3, 4
$x^T A x = 55.72$ $x^T A^{-1} x = 20.635258$

Using the SIMFIT procedure to evaluate quadratic forms allows the matrix  $A$  and vector  $x$  to be changed but two facts must be clear.

1. The dimensions of  $A$  and  $x$  must be consistent, i.e. identical.
2. Calculation of  $Q_2$  requires that  $A$  is nonsingular.

Of course, in many applications, as when estimating a Mahalanobis distance in multivariate statistics, it is also vital that the matrix  $A$  to be used is a symmetric positive definite matrix (e.g. a covariance matrix) and the vector  $x$  has a defined meaning (e.g. a difference vector) if the scalar results from such quadratic forms are to be interpreted correctly.