



A polynomial  $f(x)$  of degree  $n - 1$  has  $n$  coefficients, either  $p_i$  defined in ascending order of powers of  $x$ , or  $A_j$  defined in descending order of powers of  $x$ . It also has  $n - 1$  zeros, i.e. roots  $\alpha_i$  satisfying  $f(\alpha_i) = 0$ , so it can be expressed in terms of monomials  $x^k$  for  $k = 1, 2, \dots, n - 1$ , or factored into linear terms  $x - \alpha_i$  as follows

$$\begin{aligned} f(x) &= p_0 + p_1x + p_2x^2 + \dots + p_{n-1}x^{n-1} \\ &= A_1x^{n-1} + A_2x^{n-2} + A_3x^{n-3} + \dots + A_n \\ &= p_{n-1}(x - \alpha_1)(x - \alpha_2) \dots (x - \alpha_{n-1}). \end{aligned}$$

Numerical estimation of the zeros  $\alpha_i$  given the coefficients can be done interactively by choosing [Statistics] from the main SIMFIT menu, then [Numerical analysis] followed by selecting the options to estimate the zeros of a polynomial.

The default parameters are  $A_i = i$  for  $i = 1, 2, \dots, 5$  leading to the following results.

Zeros of $f(x) = A_1x^{n-1} + A_2x^{n-2} + \dots + A_5$		
Coefficients	Real Part	Imaginary Part
$A_1 = 1.0$	-1.49179800	
$A_2 = 2.0$	-0.80578647	-1.2229047
$A_3 = 3.0$	-0.80578647	1.2229047
$A_4 = 4.0$	0.55168546	-1.2533489
$A_5 = 5.0$	0.55168546	1.2533489
$A_6 = 6.0$		

One of the problems in estimating the roots of polynomials is when either the real or the imaginary parts are close to zero. For instance with the difficult case

$$\begin{aligned} f(x) &= (x - 1)^2(x^2 + 1) \\ &= x^4 - 2x^3 + 2x^2 - 2x + 1 \\ &= (x - 1)(x - 1)(x + i)(x - i) \end{aligned}$$

leading to the following results.

Zeros of $f(x) = A_1x^{n-1} + A_2x^{n-2} + \dots + A_5$		
Coefficients	Real Part	Imaginary Part
$A_1 = 1.0$	1.0	
$A_2 = -2.0$		-1.0
$A_3 = 2.0$		1.0
$A_4 = -2.0$	1.0	
$A_5 = 1.0$		

This example illustrates how the output from the SIMFIT routine suppresses values for very small real or imaginary components to emphasize cases with pure real roots and pure nonreal roots. However, for completeness, full details of the estimated values for all real and imaginary parts are written to the results file.