Tutorials and worked examples for simulation, curve fitting, statistical analysis, and plotting.
https://simfit.org.uk
https://simfit.silverfrost.com

SIMFIT provides two types of run test.

1. **Run test on successive signs.**
   This is used to analyze residuals from regression or, in fact, any succession of observations of a variable which can only have one of two values, say positive (+ve) or negative (-ve).

2. **Runs up or down test.**
   This is mainly used to test sequences of numbers for significant correlation, as in examining the performance of a random number generator, and is discussed elsewhere.

To be precise, the run test considered in this article is based on an application of the binomial distribution, and is used when the sequence of successes and failures (presumed in the null hypothesis to be equally likely) is of interest, not just the overall proportions. To understand the definition of runs as dealt with by this test, just consider the sequence

$$+++--++---+-$$

or alternatively

$$111001100010$$

which has twelve items with six runs, as will be clear by adding brackets like this

$$(aaa)(bb)(aa)(bbb)(a)(b).$$

Open the SIMFIT main menu, select [A/Z], then choose to run SIMFIT program **rstest**. This provides three quite separate ways to perform a run test as follows.

1. **Direct input of parameters.**
   You simply type in the number of negative and positive signs observed and the associated runs to get an analysis like the following results for the default values of 10 positives, 10 negatives, and 10 runs.

Run and sign test 1	
Number of -ve values	10
Number of +ve values	10
Number of runs	10
Probability(runs ≤ observed;	
given number of +ve, -ve values)	0.41407
Critical number for 1% significance level	5
Critical number for 5% significance level	6
Probability(runs ≤ observed;	
given number of non zero values)	0.50000
Probability(signs ≤ observed)	
(Two tail sign test statistic)	1.00000

   Note that when defining parameters in this way you will be warned if there is an inconsistency in the data supplied.

2. **Direct input of residuals.**
   Using this method a file containing the residuals is input to give results like the following for the default test file rstest.tf1.

Run and sign test 2

Number of -ve values	29
Number of +ve values	21
Number of runs	21
Probability(runs ≤ observed;	
given number of +ve, -ve values)	0.12867
Critical number for 1% significance level	17
Critical number for 5% significance level	19
Probability(runs ≤ observed;	
given number of non zero values)	0.12643
Probability(signs ≤ observed)	
(Two tail sign test statistic)	0.32224

3. **Direct input of two sequences of values.**
   Another way is to input two files containing numerical values, then allow SɪᴍFɪT to calculate the residuals as for the next results with the default test files `rstest.tf1` and `normal.tf1`.

Run and sign test 3

Number of -ve values	24
Number of +ve values	26
Number of runs	26
Probability(runs ≤ observed;	
given number. of +ve, -ve values)	0.56142
Critical number for 1% significance level	17
Critical number for 5% significance level	20
Probability(runs ≤ observed;	
given number of non zero values)	0.61228
Probability(signs ≤ observed)	
(Two tail sign test statistic)	0.88772

To emphasize the advantages of the run test over the sign test, consider a situation that a sample of ten new born babies in a hospital ward consisted of five boys and five girls? That would appear reasonable. However, what if all the boys were born first in the morning, then all the girls in the afternoon, that is, two runs? Clearly the sign test alone does not help, but the next table would confirm what most would believe intuitively: the event may not represent random sampling but could suggest the operation of other factors. In this way the run test, particularly when conditional upon the number of successes and failures, is using information from the sequence of outcomes and is therefore more powerful than the sign test alone.

Run and sign test 4

Number of -ve values	5	
Number of +ve values	5	
Number of runs	2	
Probability(runs ≤ observed;		
given number of +ve, -ve values)	0.00794	Reject H_0 at 1% significance level
Critical number for 1% sig. level	2	
Critical number for 5% sig. level	3	
Probability(runs ≤ observed;		
given number of non zero values)	0.01953	Reject H_0 at 5% significance level
Probability(signs ≤ observed)		
(Two tail sign test statistic)	1.00000	

Note that every time SɪᴍFɪT performs curve fitting it gives an analysis of goodness of fit which includes the run test to draw attention to bias in the fit resulting in too few runs caused by sections where the best-fit curve lies appreciably to one side of the data. Actually residuals from regression are not exactly normally

distributed even if the experimental errors are and the model fitted is correct, due to dependence introduced by the estimation of parameters. However, if the total numbers of data points fitted is much larger than the number of parameters estimated, this complication will not be very important.

Note that the run test in the analysis of residuals depends on a natural ordering, for instance, when the residuals are arranged to correspond to the order of a single independent variable. This is not possible if there are replicates, or several independent variables so, to use the run test in such circumstances, the residuals must be arranged in some meaningful sequence, such as the order in time of the observation, otherwise arbitrary results can be obtained by rearranging the order of residuals. The formulas used by SimFiT to calculate the statistics in the run test analysis are presented next.

Given the numbers of positive and negative residuals, the probability of any possible number of runs can be calculated by enumerating all possible arrangements. For instance, the random number of runs R given m positive and n negative residuals (redefining if necessary so that $m \leq n$) depends on whether the number of runs is even or odd as follows

$$P(R = 2k) = \frac{2\binom{m-1}{k-1}\binom{n-1}{k-1}}{\binom{m+n}{m}},$$

$$\text{or } P(R = 2k+1) = \frac{\binom{m-1}{k-1}\binom{n-1}{k} + \binom{m-1}{k}\binom{n-1}{k-1}}{\binom{m+n}{m}}.$$

Here the maximum number of runs is $2m + 1$ if $m < n$, or $2m$ if $m = n$, and $k = 1, 2, \ldots, m \leq n$. However, in the special case that $m > 20$ and $n > 20$, the probabilities of r runs can be estimated by using a normal distribution with

$$\mu = \frac{2mn}{m+n} + 1,$$

$$\sigma^2 = \frac{2mn(2mn - m - n)}{(m+n)^2(m+n-1)},$$

$$\text{and } z = \frac{r - \mu + 0.5}{\sigma},$$

where the usual continuity correction is employed.

The previous conditional probabilities depend on the values of m and n, but it is sometimes useful to know the absolute probability of R runs given $N = n + m$ nonzero residuals. There will always be at least one run, so the probability of r runs occurring depends on the number of ways of choosing break points where a sequence of residuals changes sign. This will be the same as the number of ways of choosing $r - 1$ items from $N - 1$ without respect to order, divided by the total number of possible configurations, i.e. the probability of $r - 1$ successes in $N - 1$ independent Bernoulli trials given by

$$P(R = r) = \binom{N-1}{r-1}\left(\frac{1}{2}\right)^{N-1}.$$

This is the value referred to as the probability of runs given the number of nonzero residuals in the previous tables of results.