Simfit

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The one sample t test is one of many tests designed to see if a sample can be regarded as consistent with a normal distribution but, for this test, it is also a strict requirement that the theoretical mean value is μ_0, a parameter that is known in advance from previous investigations.

To be precise, the user has a sample (i.e. vector X) of n observations

$$X = (x_1, x_2, \ldots, x_n)$$

and wishes to test if these numbers are consistent with a normal distribution where the mean μ_0 has been previously estimated with great precision from an independent very large sample. Preferably the data should cover a wide range and n should not be too small, say $n \geq 20$?

Choose `[A/Z]` from the main SIMFIT menu, open program **ttest**, select the 1-sample-t-test option, input a theoretical mean $\mu_0 = 0$, then analyze the test data set to obtain the following results.

Normal distribution test

Data: Test file `normal.tf1` with 50 pseudo-random numbers	
Shapiro-Wilks statistic W	0.9627
Significance level for W	0.1153
Conclusion: *Tentatively accept normality*	

One sample t test

Number of x-values	50
Number of degrees of freedom	49
Theoretical mean (μ_0)	0
Sample mean ($\bar{x}$)	−0.02579
Standard error of mean (SE)	0.1422
$TS = (\bar{x} - \mu_0)/SE$	−0.1814
$P(t \geq TS)$ (upper tail p)	0.5716
$P(t \leq TS)$ (lower tail p)	0.4284
p for two tailed t test	0.8568
Difference $D = \bar{x} - \mu_0$	−0.02579
Lower 95% confidence limit for D	−0.3116
Upper 95% confidence limit for D	0.2600
Conclusion: *Consider accepting equality of means*	

The analysis begins by performing a Shapiro-Wilks test to see if the sample can be regarded as normally distributed using the sample estimates for both the mean and the standard deviation. This test is less powerful than the one sample t test and, if it rejects the null hypothesis of an arbitrary normal distribution, the subsequent results can be ignored. Clearly, there is no evidence to support rejection of the hypothesis of an arbitrary normal distribution. The further analysis goes on to examine the size of the difference between the sample mean $\bar{x}$ and the theoretical mean μ_0, given the sample variance estimate, using the t distribution, and concludes that the data do appear to be normally distributed with mean close to $\mu_0 = 0$, as the two tail p value is 0.8568.

In order to appreciate the sensitivity of this test to the assumed value for μ_0, you should repeat the test using a different value for the theoretical mean, say $\mu_0 = 1$ for instance, which leads to the next results.

One sample t test	
Number of of x-values	50
Number of degrees of freedom	49
Theoretical mean (μ_0)	1
Sample mean ($\bar{x}$)	−0.02579
Standard error of mean (SE)	0.1422
$TS = (\bar{x} - \mu_0)/SE$	−7.214
$P(t \geq TS)$ (upper tail p)	1.0000
$P(t \leq TS)$ (lower tail p)	0.0000
p for two tailed t test	0.0000
Difference $D = \bar{x} - \mu_0$	−1.026
Lower 95% confidence limit for D	−1.312
Upper 95% confidence limit for D	−0.740
Conclusion: *Reject equality of means at 1% significance level*	

Obviously the Shapiro-Wilks test result is unchanged, but now the lower tail p value strongly indicates that the sample is shifted to the left of a normal distribution with $\mu = 1$, and the two tail p value, which is twice the lesser of the upper and lower tail probabilities, clearly rejects the null hypothesis $H_0 : \mu_0 = 1$.

The following graph makes it clear why $H_0 : \mu_0 = 1$ was rejected.