It is frequently necessary to compare sample variances for equality, and there are also numerous other applications for examining variance estimates, such as analysis of variance or excess variance, where a test statistic is required. The F distribution arises naturally in such contexts.

1 Definitions

If there are two independent random chi-square variables: U with m degrees of freedom, and V with n degrees of freedom, then the ratio of these divided by their respective degrees of freedom as in

$$F = \frac{U/m}{V/n}$$

defines the F distribution with m and n degrees of freedom. The expectation is given by

$$E(F) = \frac{n}{n-2}$$

when $n > 2$ and, rather than performing upper, lower, or two tail tests, the ratio is often inverted in some applications so that the numerator is always greater than the denominator, resulting in values of test statistics $F \geq 1$.

There is a good reason for this. Because of a special property of this distribution, lower tail percentiles are readily available from upper tail percentiles and vice versa. To see this, note that

$$
\begin{aligned}
P(F < F_{.05}) &= 0.05 \\
&= 1 - P(F > F_{.05}) \\
&= 1 - P\left(\frac{1}{F} < \frac{1}{F_{.05}}\right)
\end{aligned}
$$

where $F_{.05}$ is the 5% critical point for the distribution of F, so that $1/F_{.05}$ is the 95% critical point for the distribution of $1/F$. Obviously, as F is distributed with m and n degrees of freedom, then $1/F$ is F distributed with n and m degrees of freedom.

2 Using the F distribution

Choose [A/Z] from the main SimFit menu and open program **ftest** when the following options will be available.

```
Input: current parameters m and n
Input: x-value then output pdf(x)
Input: x-value then output cdf(x)
Input: alpha then calculate x-critical
Input: sample then test distributed F(m,n)
Input: sums of squares then perform F test
Do 1,2,3-way Analysis of Variance
Calculate non-central F distribution values
```

Selecting the first option and choosing $m = 10$ and $n = 12$ displays the following plot.

F distribution: m = 10, n = 12
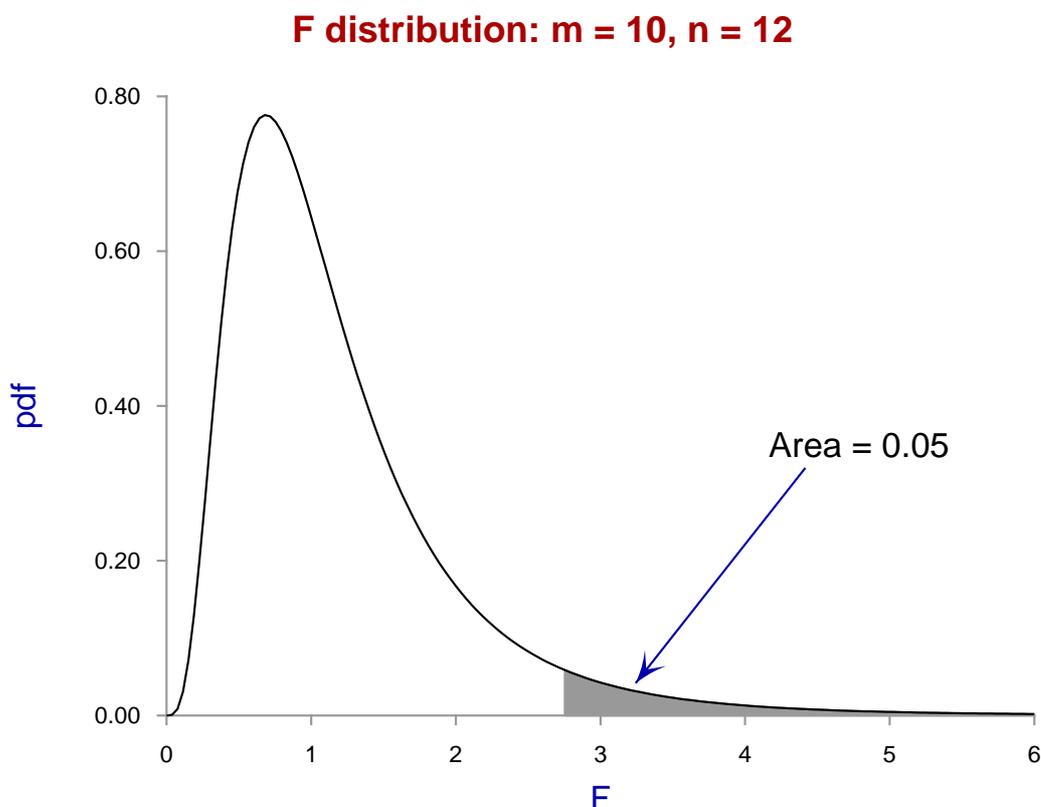
When using the F distribution, SIMF$_I$T will usually return the probability of a test statistic, say TS, being acceptable or not in the following way

$$P(TS \geq F) = 0.6742$$

or similar. If TS is sufficiently large to fall in the upper tail for the corresponding F distribution, such as in the region shaded in the above figure, there would then be a further message indicating the possibility for rejecting the null hypothesis at the 5% significance level, i.e. $p \leq 0.05$ or, in even more extreme cases, $p \leq 0.01$ In other words, $P(TS \geq F)$ is simply the area in the upper tail beyond the test statistic TS, i.e. the significance level for rejection of the null hypothesis.